Single-frame multichannel blind deconvolution by nonnegative matrix factorization with sparseness constraints

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Received July 13, 2005; accepted August 8, 2005

Single-frame multichannel blind deconvolution is formulated by applying a bank of Gabor filters to a blurred image. The key observation is that spatially oriented Gabor filters produce sparse images and that a multichannel version of the observed image can be represented as a product of an unknown nonnegative sparse mixing vector and an unknown nonnegative source image. Therefore a blind-deconvolution problem is formulated as a nonnegative matrix factorization problem with a sparseness constraint. No *a priori* knowledge about the blurring kernel or the original image is required. The good experimental results demonstrate the viability of the proposed concept. © 2005 Optical Society of America

OCIS codes: 070.6020, 100.1830, 100.3020, 100.3190.

The goal of blind deconvolution (BD) is to reconstruct the original image from an observation degraded by spatially invariant blurring and noise. Neglecting the noise term, the process is modeled as a convolution of a point-spread function (PSF) $h(x,y)$ with an original source image $f(x,y)$ as follows:

$$g(x,y) = \sum_{s=-M}^{M} \sum_{t=-M}^{M} h(s,t) f(x+s, y+t), \qquad (1)$$

where M denotes the PSF support size. If the PSF is known, a number of algorithms are available to reconstruct original image $f(x,y)$.[1] However, it is not always possible to measure or obtain information about a PSF, which is why BD algorithms are important. A comprehensive comparison of BD algorithms is given in Ref. 1. BD approaches can be divided into those that estimate the blurring kernel $h(s,t)$ first and then restore the original image by some of the nonblind methods[1] and those that estimate the original image $f(x,y)$ and the blurring kernel simultaneously. To estimate the blurring kernel, one must either know or estimate a support size. Also, quite often *a priori* knowledge about the nature of the blurring process is assumed to be available when an appropriate parametric model of the blurring process is used.[2] It is not always possible to know the characteristics of the blurring process. Methods that estimate the blurring kernel and the original image simultaneously use either statistical or deterministic prior knowledge of the original image, the blurring kernel, and the noise,[2] which leads to a computationally expensive maximum-likelihood estimation that is usually implemented by an expectation maximization algorithm. In addition, exact distribution of the original image required by a maximum-likelihood algorithm are usually unknown. To overcome these difficulties an approach was proposed in Ref. 3 that was based on a quasi-maximum likelihood with an approximation of the probability-density function. It was, however, assumed that the original image has a sparse or super-Gaussian distribution, which is generally not true because image distributions are mostly sub-Gaussian. To overcome that difficulty it was proposed in Ref. 3 to apply a so-called sparsifying transform to a blurred image. However, the design of such a transform requires knowledge of at least the typical class of image to which the original image belongs, in which case training data can be used to design the sparsifying transform. Multivariate data analysis methods, such as independent component analysis (ICA),[4] might be used to solve a BD problem as a blind source-separation problem, where unknown blurring would be absorbed into what is known as a mixing matrix. However, the multichannel image required by ICA is not always available. Even if it were, it would require the blurring kernel to be nonstationary, as it is for blurring caused by atmospheric turbulence[5] but is not for out-of-focus blur, for example. For all the reasons discussed above, an approach to single-frame multichannel BD that requires a minimum of *a priori* information about the blurring process and the original image would be of great interest. One such approach was proposed in Ref. 6. It was based on a bank of two-dimensional (2-D) Gabor filters[7] used because of their ability to produce multichannel filtering and to decompose an input image into sparse images containing intensity variations over a narrow range of frequency and orientation, which is characteristic of the visual cortices of some mammals.[7] The concept is illustrated in Fig. 1; 2-D Gabor filters are shown in Fig. 2 for two spatial frequencies and four orientations. The top two rows of Fig. 2 show real and imaginary parts of 2-D Gabor filters for one spatial frequency; the bottom

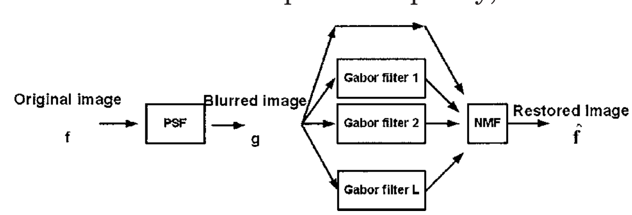

Fig. 1. Image restoration by nonnegative matrix factorization and 2-D Gabor filters.

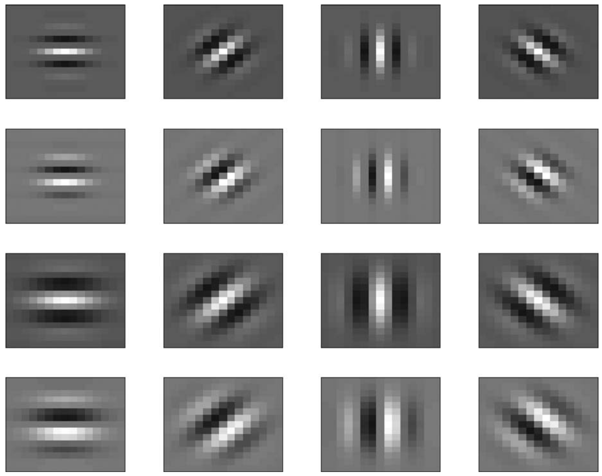

Fig. 2. Gabor filters for two spatial frequencies and four orientations. The top two rows show real and imaginary parts of 2-D Gabor filters for one spatial frequency; the bottom two rows, for another spatial frequency. Each column shows one of the four orientations.

two rows, those for another spatial frequency. Each column shows one of the four orientations. The key insight in Ref. 6 was that the original image $f(x+s,y+t)$ can be approximated by a Taylor series expansion about $f(x,y)$, giving $f(x+s,y+t)=f(x,y)+sf_x(x,y)+tf_y(x,y)+\ldots$. This enables us to rewrite Eq. (1) as

$$g(x,y) = a_1 f(x,y) + a_2 f_x(x,y) + a_3 f_y(x,y) + \ldots, \quad (2)$$

where $a_1=\Sigma_{s=-M}^{M}\Sigma_{t=-M}^{M} h(s,t)$, $a_2=\Sigma_{s=-M}^{M}\Sigma_{t=-M}^{M} sh(s,t)$, $a_3=\Sigma_{s=-M}^{M}\Sigma_{t=-M}^{M} th(s,t)$, and f_x and f_y are spatial derivatives in the x and y directions, respectively. When Gabor filters are applied to a blurred image, a new set of observed images is obtained as

$$g_l(x,y) = a_{l1} f(x,y) + a_{l2} f_x(x,y) + a_{l3} f_y(x,y) + \ldots, \quad (3)$$

where $a_{l1}=\Sigma_{s=-M}^{M}\Sigma_{t=-M}^{M} h_l'(s,t)$, $a_{l2}=\Sigma_{s=-M}^{M}\Sigma_{t=-M}^{M} sh_l'(s,t)$, and $a_{l3}=\Sigma_{s=-M}^{M}\Sigma_{t=-M}^{M} th_l'(s,t)$, where $h_l'(s,t)$ represents convolution of the appropriate lth Gabor filter with $h(s,t)$, which leads to this multichannel representation:

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}^{\mathrm{T}} \\ \mathbf{g}_1{}^{\mathrm{T}} \\ \vdots \\ \mathbf{g}_L{}^{\mathrm{T}} \end{bmatrix} \cong \begin{bmatrix} a_1 & a_2 & a_3 \\ a_{11} & a_{12} & a_{13} \\ \vdots & \vdots & \vdots \\ a_{L1} & a_{L2} & a_{L3} \end{bmatrix} \begin{bmatrix} \mathbf{f}^{\mathrm{T}} \\ \mathbf{f}_x{}^{\mathrm{T}} \\ \mathbf{f}_y{}^{\mathrm{T}} \end{bmatrix} = \mathbf{AF}, \quad (4)$$

to which ICA algorithms can be applied to extract source image **f**. The lexicographical (vector) representation assumed in expression (4) for original image **f** and observed images **g** and $\mathbf{g}_1$ is obtained by a row-stacking procedure. As can be seen, no *a priori* information about a blurring kernel is assumed so far. There is, however, a critical condition that must hold for the source image for the ICA algorithm to work. Images **f**, $\mathbf{f}_x$, and $\mathbf{f}_y$ must be statistically independent. In general they are not, as was already observed in Ref. 8. Here two key contributions are made that will enable us to use a single-frame multichannel representation and make no assumption about the statistical nature of the source image. First, we observe that both $\mathbf{f}_x$ and $\mathbf{f}_y$ can be written as a convolution of the original image $f(x,y)$ with some directional filters, giving $f_x(x,y)\cong\Sigma_{s=-N}^{N}\Sigma_{t=-N}^{N} h_x(s,t)f(x+s,y+t)$ and $f_y(x,y)\cong\Sigma_{s=-N}^{N}\Sigma_{t=-N}^{N} h_y(s,t)f(x+s,y+t)$, where N represents the order of the directional filters. Obviously, for large N, directional filters will well approximate spatial derivatives along the x and y directions. Because N does not itself play a role in a BD algorithm, we can assume that it is arbitrarily large. Consequently, the approximation symbol may be replaced by an equality symbol. By using the same expansion for $f(x+s,y+t)$ as for Eq. (2) we obtain

$$\begin{aligned} f_x(x,y) = {} & \left[\sum_{s=-N}^{N}\sum_{t=-N}^{N} h_x(s,t)\right] f(x,y) \\ & + \left[\sum_{s=-N}^{N}\sum_{t=-N}^{N} sh_x(s,t)\right] f_x(x,y) \\ & + \left[\sum_{s=-N}^{N}\sum_{t=-N}^{N} th_x(s,t)\right] f_y(x,y) + \ldots, \end{aligned}$$

$$\begin{aligned} f_y(x,y) = {} & \left[\sum_{s=-N}^{N}\sum_{t=-N}^{N} h_y(s,t)\right] f(x,y) \\ & + \left[\sum_{s=-N}^{N}\sum_{t=-N}^{N} sh_y(s,t)\right] f_x(x,y) \\ & + \left[\sum_{s=-N}^{N}\sum_{t=-N}^{N} th_y(s,t)\right] f_y(x,y) + \ldots. \end{aligned} \quad (5)$$

Because h_x and h_y are directional filters, the second and third terms in the expansions of $f_x(x,y)$ and $f_y(x,y)$ in Eqs. (5) will be either equal to zero or small compared with the first term. This enables us to write Eq. (2) as $g(x,y)\cong(a_1+a_2a_1{}^x+a_3a_1{}^y)f(x,y)\cong\tilde{a}_1 f(x,y)$, where it follows from Eqs. (5) that $a_1{}^x=\Sigma_{s=-N}^{N}\Sigma_{t=-N}^{N} h_x(s,t)$ and $a_1{}^y=\Sigma_{s=-N}^{N}\Sigma_{t=-N}^{N} h_y(s,t)$. Following the same logic, Eq. (3) can be rewritten as

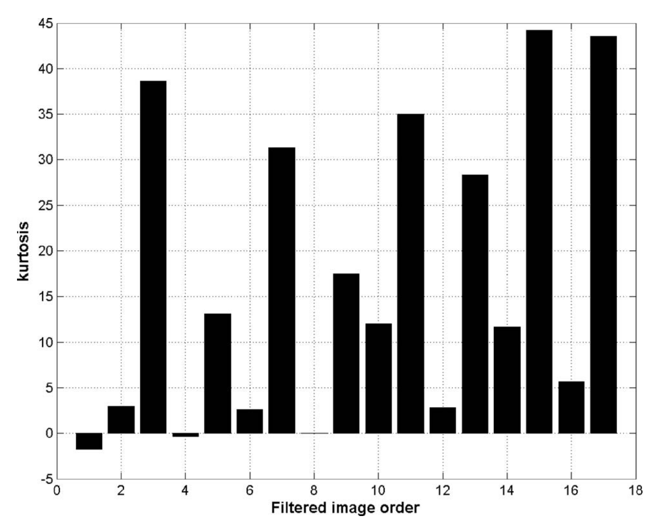

Fig. 3. Kurtosis of the blurred and Gabor-filtered images.

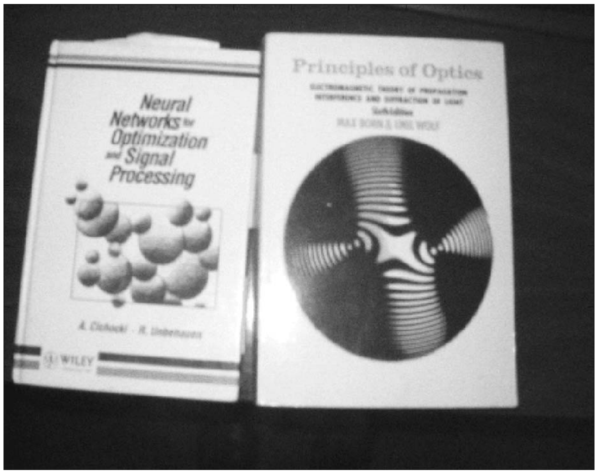

Fig. 4. Defocused image.

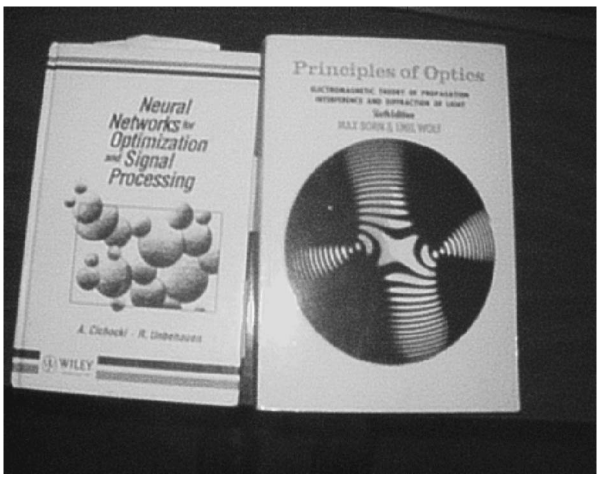

Fig. 5. Reconstructed image.

$g_l(x,y) \cong \tilde{a}_{l1} f(x,y)$, and matrix equation (4) becomes

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}^{\mathrm{T}} \\ \mathbf{g}_1{}^{\mathrm{T}} \\ \vdots \\ \mathbf{g}_L{}^{\mathrm{T}} \end{bmatrix} \cong \begin{bmatrix} \tilde{a}_1 \\ \tilde{a}_{21} \\ \vdots \\ \tilde{a}_{L1} \end{bmatrix} [\mathbf{f}^{\mathrm{T}}] = \tilde{\mathbf{a}} \mathbf{f}^{\mathrm{T}}, \tag{6}$$

which suggests the existence of only a source image in the linear image observation model. In expression (6), T denotes a transpose. The second key insight is that spatially oriented Gabor filters will produce images with sparse (super-Gaussian) distributions and, because source image $\mathbf{f}$ is mostly sub-Gaussian, an unknown mixing vector $\tilde{\mathbf{a}}$ must be sparse. Because $\tilde{\mathbf{a}}$ and $\mathbf{f}$ are nonnegative we are able to formulate a BD problem as a nonnegative matrix factorization (NMF) problem with a sparseness constraint.[9] Estimates of mixing vector $\tilde{\mathbf{a}}$ and source image $\mathbf{f}$ are obtained as a solution of the minimization problem:

$$(\hat{\tilde{\mathbf{a}}}, \hat{\mathbf{f}}) = \min \sum_{i,j} [G_{ij} - (\hat{\tilde{\mathbf{a}}} \hat{\mathbf{f}}^{\mathrm{T}})_{ij}]^2 \tag{7}$$

under sparseness constraint S_a imposed on the estimate of mixing vector $\tilde{\mathbf{a}}$. S_a is a number from 0 to 1, with 1 meaning that all components of vector $\tilde{\mathbf{a}}$ are small and 0 meaning the opposite.[9] Refer to Ref. 9 for detailed descriptions of the computational steps associated with the NMF algorithm and to http://www.cs.helsinki.fi/patrik.hoyer/ for its MATLAB implementation, which was used in the experiment reported. Refer to Ref. 9 and references therein for more details about NMF algorithms. Because this is a deterministic approach, no assumption about the statistical nature of either blur or the source image is required. Only a sparseness constraint must be imposed on unknown mixing vector $\tilde{\mathbf{a}}$. The first coefficient in $\tilde{\mathbf{a}}$ can initially be approximated by 1, because it represents the original blurring process. The rest of the coefficients can initially be set to 0 because they correspond to sparse images. Therefore the initial value of the unknown mixing vector is set to $\tilde{\mathbf{a}}^{(0)} = [1\ 0\ 0 \ldots 0]^{\mathrm{T}}$. A sparseness constraint S_a must be defined for the NMF algorithm. To obtain a truly unsupervised image restoration algorithm we estimate S_a from multichannel image $\mathbf{G}$ as a ratio between the number of sparse images $\mathbf{L}_s$ and the overall number of images $\mathbf{L}+1$. To estimate $\mathbf{L}_s$, kurtosis κ of each image in $\mathbf{G}$ is estimated. Image $\mathbf{g}_l$ is considered to be sparse if $\kappa(\mathbf{g}_l) > \delta$. In the experiments I set $\delta = 0.2$. This completes the derivation of the single-frame multichannel BD algorithm that is defined without use of any *a priori* information about the blurring process or the original image. I comment here that a sparseness constraint might be imposed on the source image too if it is known that a particular imaging modality will generate a sparse image, as could be the case in astronomy or microscopy. In Fig. 3 the estimated kurtosis value of the blurred image shown in Fig. 4 is given in column 1 and the Gabor-filter-generated images by columns 2–17. The estimated sparseness constraint was $S_a = 0.82$. The blurred image was obtained by a defocused digital camera. The reconstructed image is shown in Fig. 5.

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